Coverability Synthesis in Parametric Petri Nets

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LS2N, Nantes

Why Introducing Parameters ?

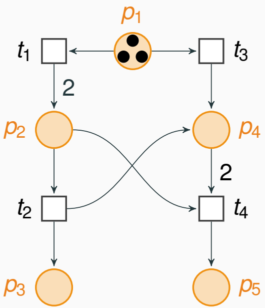
- modeling arbitrary large amount of processes
- modeling unspecified aspect of the environment
- provide a higher level of abstraction

Outline

1. Petri Nets
2. Parametric Petri Nets
3. Undecidability of the Generic Case
4. From Monotonicity in PPNs to Synthesis
5. Decidability in DistinctT-PPNs
6. What about Complexities ?
7. EXPSPACE completeness of U-Cov in PreT-PPNs
8. Establishing Frontiers - Work In Progress
9. Conclusion

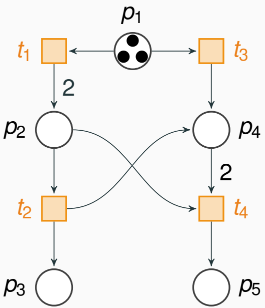
Petri Nets

Definition



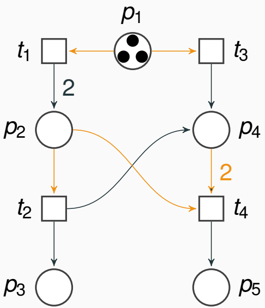
- P

Definition



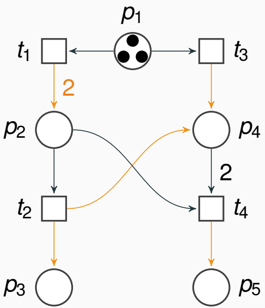
- P
- T

Definition

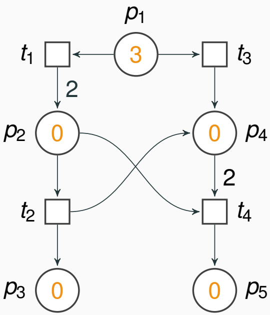


- P
- T
- Pre : $P \times T \to \mathbb{N}$

Definition

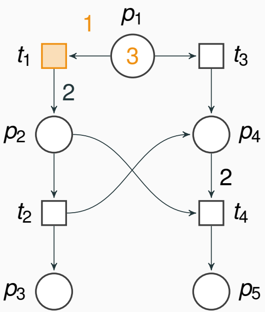


- P
- T
- Pre : $P \times T \to \mathbb{N}$
- Post : $P \times T \to \mathbb{N}$



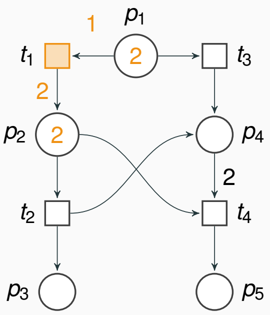
- P
- T
- Pre : $P \times T \to \mathbb{N}$
- Post : $P \times T \to \mathbb{N}$
- $m : P \to \mathbb{N}^k$

Semantics



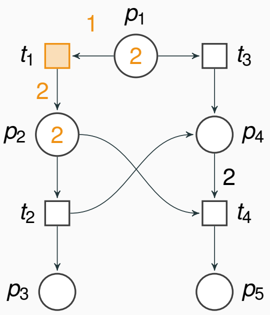
- $m \geq Pre(t_1)$

Semantics



- $m \geq Pre(t_1)$
- $m' = m - Pre(t_1) + Post(t_1)$

Semantics



- $m \geq Pre(t_1)$
- $m' = m - Pre(t_1) + Post(t_1)$
- $\Leftrightarrow m \xrightarrow{t_1} m'$

Properties

Reachability

Let $\mathcal{S} = (\mathcal{N}, m_0)$, where $\mathcal{N} = (P, T, Pre, Post)$, a marking m of $\mathbb{N}^P$ is reachable in $\mathcal{S}$ if $m_0 \xrightarrow{*} m$.

The reachability set $\mathcal{RS}(\mathcal{S})$ of $\mathcal{S}$ is the set of all reachable markings of $\mathcal{S}$.

Coverability

Let $\mathcal{S} = (\mathcal{N}, m_0)$, where $\mathcal{N} = (P, T, Pre, Post)$, and m a marking of $\mathbb{N}^P$, m is coverable in $\mathcal{S}$ if $\exists m' \in \mathcal{RS}(\mathcal{S}), m' \geq m$.

we write $\texttt{cov}(\mathcal{SP}, m)$

Some Precisions on Coverability

The coverability set $CS(\mathcal{S}) = \{m \mid \texttt{cov}((\mathcal{S}), m)\}$

Coverability is decidable in PNs [Karp and Miller, 1969].

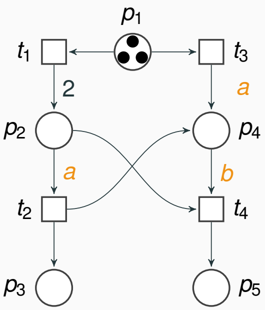
We can compute the basis s.t.

$CS(\mathcal{S}) = \{m \mid \exists x \in \mathcal{BCS}(\mathcal{N}, m_0),\ m \leq x\}$

with $\mathcal{BCS}(\mathcal{N}, m_0) \subseteq \mathbb{N}_\omega^{|P|}$

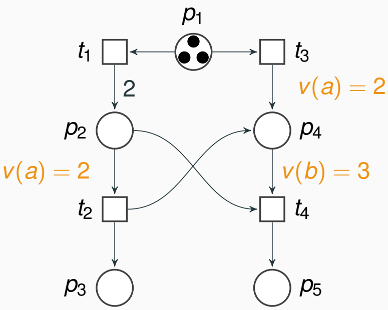
Parametric Petri Nets

Definition



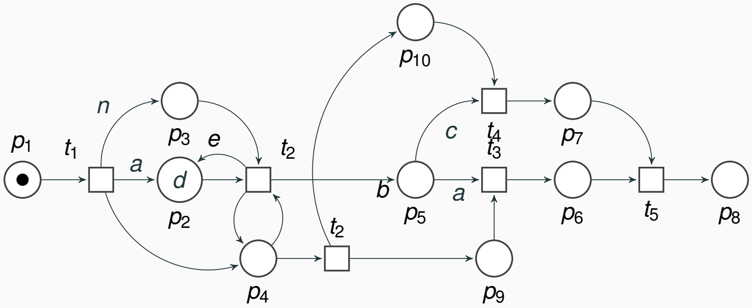
- P
- T
- Pre : $P \times T \to \mathbb{N} \cup \mathbb{P}$
- Post : $P \times T \to \mathbb{N} \cup \mathbb{P}$
- $\mathbb{P}$

Definition



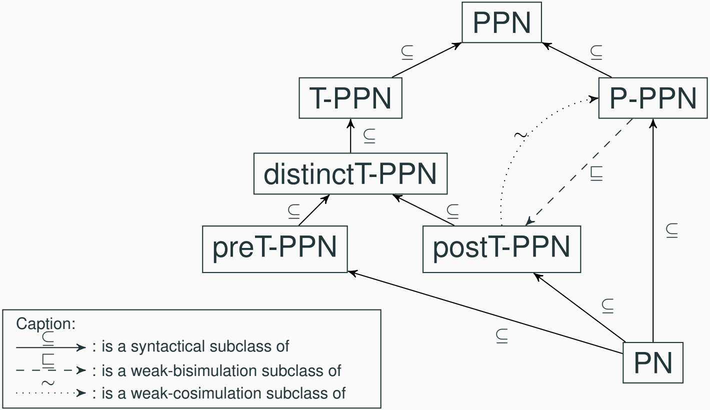
- v(P, T, Pre, Post)

Example : Financial Loan



- n = number of maturities
- a = amount loaned
- b = repayment
- e = income
- c = interest earned

Hierarchy of Parametric Petri Nets [David et al., 2015]



Properties

coverability-Existence problem

Is there a valuation $\nu \in \mathbb{N}^{Par}$ s.t. $\mathtt{cov}(v(\mathcal{SP}), m)$?

coverability-Universality problem

Is $\mathtt{cov}(v(\mathcal{SP}), m)$ true for each $\nu \in \mathbb{N}^{Par}$?

coverability-Synthesis problem

Compute all the valuations v, such that $\mathtt{cov}(v(\mathcal{SP}), m)$ is true.

$$\mathcal{CV}(\mathcal{S}, m) = \{v \in \mathbb{N}^{\mathbb{P}} \mid \mathtt{cov}(v(\mathcal{SP}), m) = true\}$$

	$\mathscr{U}$-problem		$\mathscr{E}$-problem	
	Reach.	Cov.	Reach.	Cov.
preT-PPN	?	?	?	?
postT-PPN	?	?	?	?
PPN	?	?	?	?
distinctT-PPN	?	?	?	?
P-PPN	?	?	?	?

Undecidability of the Generic Case

Undecidability in PPN

- The $\mathscr{E}$-coverability problem for PPN is undecidable.
- The $\mathscr{U}$-coverability problem for PPN is undecidable.

2-Counters Machine

- two counters c_1, c_2,
- states $P = \{p_0, ...p_m\}$, a terminal state labelled *halt*
- finite list of instructions $I_1, ..., I_s$ among the following list:
  - increment a counter
  - decrement a counter
  - check if a counter equals zero

Counters are assumed non negative.

Proof Preliminaries: Example of 2-Counters Machine

$$p_1.\ C_0 := C_0 + 1; goto\ p_2;$$

$$p_2.\ C_1 := C_1 + 1; goto\ p_1;$$

instructions sequence:

$(p_1, C_0 = 0, C_1 = 0)$

$\rightarrow (p_2, C_0 = 1, C_1 = 0)$

$\rightarrow (p_1, C_0 = 1, C_1 = 1)$

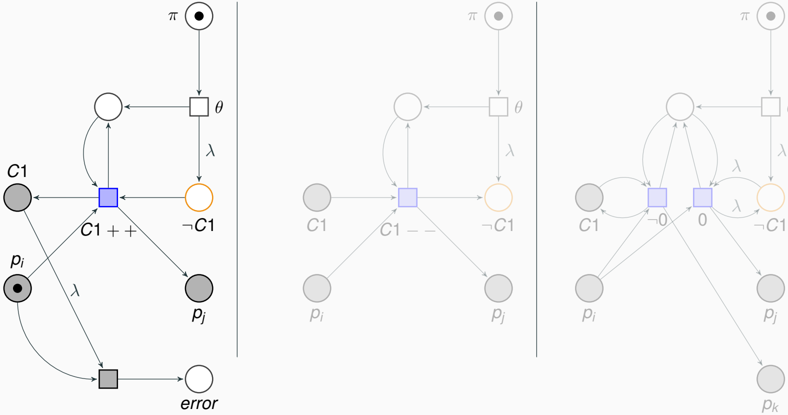
$\rightarrow (p_2, C_0 = 2, C_1 = 1)$

$\rightarrow$ ...

Proof Sketch: Simulation of a 2-Counters Machine

- halting problem (whether state *halt* is reachable) can be reduced to $\mathscr{E}$-cov
- counters boundedness problem (whether the counters values stay in a finite set) can be reduced to $\mathscr{U}$-cov
- halting problem and counters boundedness problem are undecidable

Proof Construction: Simulation of Instructions: $m(C1) + m(\neg C1) = \lambda$

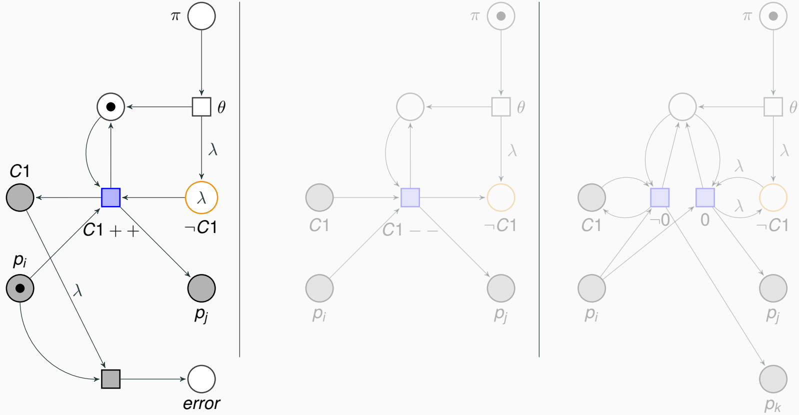


incrementation of a counter

decrementation of a counter

zero test of a counter

Proof Construction: Simulation of Instructions: $m(C1) + m(\neg C1) = \lambda$



incrementation of a counter

decrementation of a counter

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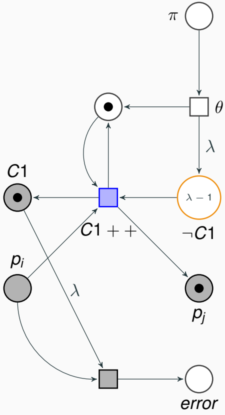
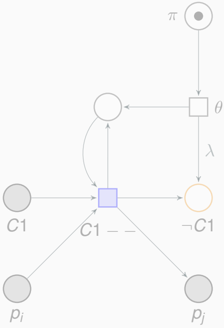
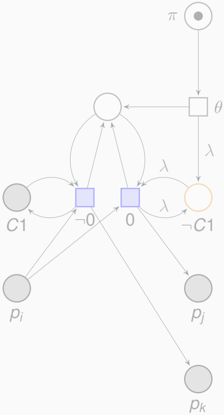
Proof Construction: Simulation of Instructions: $m(C1) + m(\neg C1) = \lambda$



incrementation of a counter

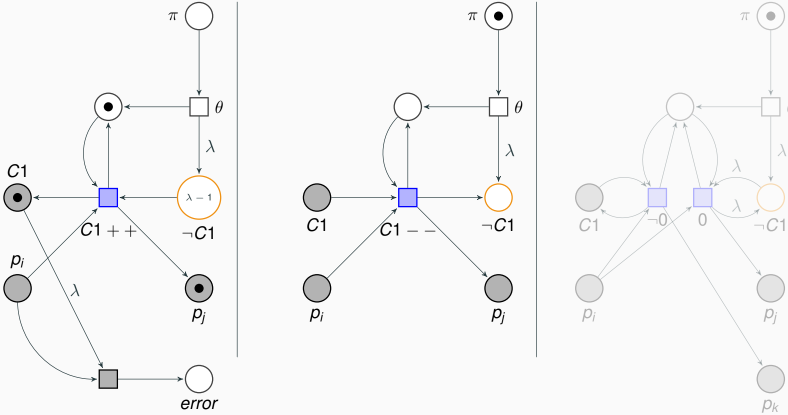


decrementation of a counter



zero test of a counter

Proof Construction: Simulation of Instructions: $m(C1) + m(\neg C1) = \lambda$

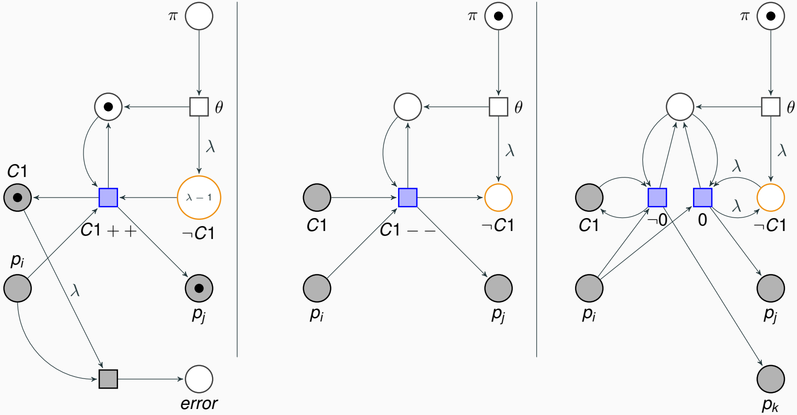


incrementation of a counter

decrementation of a counter

zero test of a counter

Proof Construction: Simulation of Instructions: $m(C1) + m(\neg C1) = \lambda$



incrementation of a counter

decrementation of a counter

zero test of a counter

- $\mathcal{M}$ halts iff there exists a valuation ν such that $\nu(\mathcal{S}_{\mathcal{M}})$ covers the corresponding p_{halt} place.
- the counters are unbounded along the instructions sequence of $\mathcal{M}$ iff for each valuation ν, $\nu(\mathcal{S}_{\mathcal{M}})$ covers the *error state*.

Let us Summarise

	$\mathscr{U}$-problem		$\mathscr{E}$-problem	
	Reach.	Cov.	Reach.	Cov.
preT-PPN	?	?	?	?
postT-PPN	?	?	?	?
PPN	?	U	?	U
distinctT-PPN	?	?	?	?
P-PPN	?	?	?	?

Let us Summarise

	$\mathscr{U}$-problem		$\mathscr{E}$-problem	
	Reach.	Cov.	Reach.	Cov.
preT-PPN	?	?	?	?
postT-PPN	?	?	?	?
PPN	U	U	U	U
distinctT-PPN	?	?	?	?
P-PPN	?	?	?	?

From Monotonicity in PPNs to Synthesis

$$\mathbb{N}_\omega = \mathbb{N} \cup \{\omega\} \text{ where } \begin{cases} \forall n \in \mathbb{N},\ n + \omega = \omega \\ \omega - n = \omega \text{ and } \omega \leq \omega \\ \forall n \in \mathbb{N},\ n < \omega \end{cases}$$

$\leq$ is the qo on $\mathbb{N}^k$ component-wise

Preliminaries on Closed Sets

U is an Upward Closed Set

$\forall x \in U, \forall y \in \mathbb{N}^k$ s.t. $x \leq y$ then $y \in U$

Upward Closure

$\uparrow u = \{m \in \mathbb{N}^k \mid u \leq m\}$
$\uparrow U = \bigcup_{u \in U} \uparrow u$

Representation

Given U upward closed
$\exists F$ finite set of $\mathbb{N}^k$, s.t. $U = \uparrow F$

D is an Downward Closed Set

$\forall x \in D, \forall y \in \mathbb{N}^k$ s.t. $y \leq x$ then $y \in D$

Downward Closure

$\downarrow d = \{m \in \mathbb{N}^k \mid m \leq d\}$
$\downarrow D = \bigcup_{d \in D} \downarrow d$

Representation

Given D downward closed
$\exists F$ finite set of $\mathbb{N}^k_\omega$ s.t. $D = \mathbb{N}^k \cap \downarrow F$

example : $CS(\mathcal{S}) = \downarrow \mathcal{RS}(\mathcal{S})$
Stable by Union, Intersection
$\mathbb{N}^k \setminus D$ is upward closed
$\mathbb{N}^k \setminus U$ is downward closed

PreT-PPNs

Let $\mathcal{S} = (\mathcal{N}, m_0)$ be a marked preT-PPN.

Intuitively, decreasing the valuation leads to a more permissive firing condition.

Monotonicity

w a transitions sequence s.t. $m_0 \xrightarrow{w} m$ in $v(\mathcal{S})$.

Then for any valuation $v' \leq v$, $m_0 \xrightarrow{w} m'$ in $v'(\mathcal{S})$ with $m' \geq m$.

PreT-PPNs

Structure of the Synthesis Set

Given a marked preT-PPN $\mathcal{S}$ and a marking m,$\mathcal{CV}(\mathcal{S}, m)$ is downward closed.

Corollary

E-cov is decidable in preT-PPNs.

proof: evaluate parameters to 0.

PostT-PPNs

Let $\mathcal{S} = (\mathcal{N}, m_0)$ be a marked postT-PPN.

Intuitively, firing the same parametric transition while increasing the valuation leads to greater (and thus more permissive) markings.

Monotonicity

w a transitions sequence s.t. $m_0 \xrightarrow{w} m$ in $v(\mathcal{S})$.

Then for any valuation $v' \geq v$, $m_0 \xrightarrow{w} m'$ in $v'(\mathcal{S})$ with $m' \geq m$.

Structure of the Synthesis Set

Given a marked postT-PPN $\mathcal{S}$ and a marking m,$\mathcal{CV}(\mathcal{S}, m)$ is upward closed.

Corollary

U-cov is decidable in postT-PPNs.

By weak bisimulation, U-cov is decidable in P-PPNs.

proof: evaluate parameters to 0.

Let us Summarize

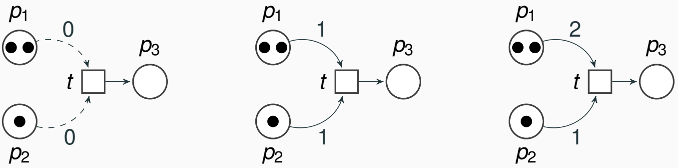
	$\mathscr{U}$-problem		$\mathscr{E}$-problem	
	Reach.	Cov.	Reach.	Cov.
preT-PPN	?	?	?	D
postT-PPN	?	D	?	?
PPN	U	U	U	U
distinctT-PPN	?	?	?	?
P-PPN	?	D	?	?

Limit for Reachability

The decidability of reachability is more complex.

increasing the valuation for preT-PPN (resp. postT-PPN)
$\Rightarrow$ disable (resp. enable) transitions
$\Leftrightarrow$ disable (resp. enable) the coverability of a marking
but
$\not\Rightarrow$ access exact number of tokens involved, *i.e.* reachability.

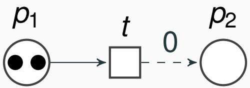
preT-PPNs



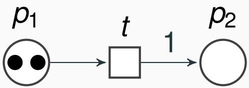
(a) 0-instance (b) 1-instance (c) 2, 1-instance

(a) $CS_0 = \downarrow (2, 1, \omega)$
(b) $CS_1 = \downarrow \{(2, 1, 0), (1, 0, 1)\} \subseteq CS_0$
(c) $CS_{2,1} = \downarrow \{(2, 1, 0), (0, 0, 1)\} \subseteq CS_1$
but
(a) $RS_0 = \{(2, 1, n) | n \in \mathbb{N}\}$
(b) $RS_1 = \{(2, 1, 0), (1, 0, 1)\} \not\subseteq RS_0$
(c) $RS_{2,1} = \{(2, 1, 0), (0, 0, 1)\} \not\subseteq RS_1$

POSTT-PPNS



(d) 0-instance



(e) 1-instance

(a) $CS_0 = \downarrow(2,0) \subseteq CS_1$
(b) $CS_1 = \downarrow\{(2,0),(1,1),(0,2)\}$
but
(a) $RS_0 = \{(2,0),(1,0),(0,0)\} \not\subseteq RS_1$
(b)$RS_1 = \{(2,0)(1,1)(0,2)\} \not\subseteq RS_0$

Computation of Closed Sets

[Goubault-Larrecq, 2009]

Given $U = \uparrow F$, we can compute F' such that $\mathbb{N}^k \setminus U = \downarrow F'$ and vice versa

Valk and Jantzen [Valk and Jantzen, 1985]

Given $U \subseteq \mathbb{N}^k$ upward closed, we can compute F
$\Leftrightarrow \forall v \in \mathbb{N}_\omega^k, \downarrow v \cap U = \emptyset$ is decidable
$\Leftrightarrow \forall v \in \mathbb{N}_\omega^k, \downarrow v \cap \mathbb{N}^k \subseteq \neg U$ is decidable

Adapting Valk and Jantzen condition

PreT-PPNs

we can compute a finite representation of the coverability synthesis set in preT-PPNs
iff universal coverability is decidable in preT-PPNs

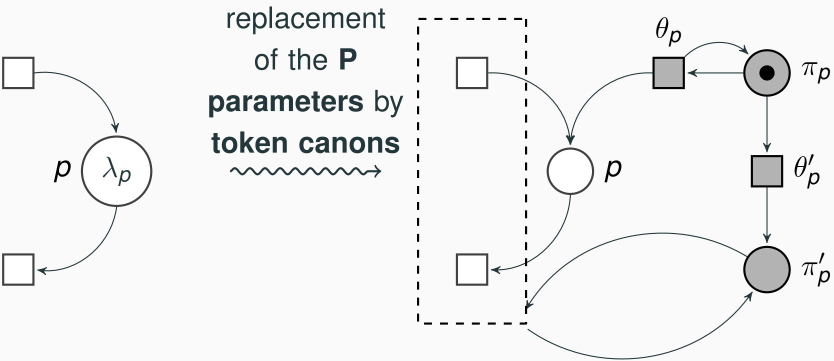
PostT-PPNs

we can compute a finite representation of the coverability synthesis set in postT-PPNs
iff existential coverability is decidable in postT-PPNs

Problems equivalent to Synthesis

	$\mathscr{U}$-problem		$\mathscr{E}$-problem	
	Reach.	Cov.	Reach.	Cov.
preT-PPN	?	?	?	D
postT-PPN	?	D	?	?
PPN	U	U	U	U
distinctT-PPN	?	?	?	?
P-PPN	?	D	?	?

DECIDABILITY OF E-COV IN POSTT-PPNS



Decidability Results

E-reach is decidable in P-PPNs.

E-cov is decidable in P-PPNs.

By weak cosimulation, E-cov is decidable in postT-PPNs.

Problems Needed for Synthesis - Some Progress !

	$\mathscr{U}$-problem		$\mathscr{E}$-problem	
	Reach.	Cov.	Reach.	Cov.
preT-PPN	?	?	?	D
postT-PPN	?	D	?	D
PPN	U	U	U	U
distinctT-PPN	?	?	?	?
P-PPN	?	D	D	D

Intuition for U-cov in PreT-PPNs

If a marking is universally coverable, two main possibilities:

- we can either reach this marking without using any parametric transition, and then the corresponding run works for any valuation,
- or we need at least one parametric transition.
  Since there is an infinite number of valuations and a finite number of parametric transitions
  $\Rightarrow$ at least one such transition must be used, as the first parametric transition in the run, for an infinite number of valuations
  $\Rightarrow$ the input places of its parametrics arcs are not bounded.

Intermediate Lemma for U-Cov in PreT-PPNs

$\mathcal{N}_p$ denote the Petri net obtained from $\mathcal{N}$ by removing all parametric transitions.

Universal Coverability

A marking m is universally coverable in $\mathcal{S}$ iff

1. m is coverable in $(\mathcal{N}_p, m_0)$ or
2. there exists $z \in \mathcal{BCS}(\mathcal{N}_p, m_0)$ such that $\omega(z) \neq \emptyset$ and $m_{|\mathbb{N}(z)}$ is universally coverable in $(\mathcal{N}_{|\mathbb{N}(z)}, z_{|\mathbb{N}(z)})$

univCov(m_0, a preT-PPN $\mathcal{N}$, m to cover)

- if $\mathrm{cov}((\mathcal{N}_p, m_0), m)$ return true
- else if $\forall z \in BCS(\mathcal{N}_p, m_0), \omega(z) = \emptyset$ then return false
- else let *achieved* = false.
  While achieved == false,
  pick an element $z \in BCS(\mathcal{N}_p, m_0)$ such that $\omega(z) \neq \emptyset$
  achieved = *achieved* or univCov($z_{|\mathbb{N}(z)}, \mathcal{N}_{|\mathbb{N}(z)}, m_{|\mathbb{N}(z)}$)
  End While
  return achieved

Decidability of U-cov in preT-PPNs

U-cov is decidable in preT-PPNs.

Problems Needed for Synthesis - More Progress !

	$\mathscr{U}$-problem		$\mathscr{E}$-problem	
	Reach.	Cov.	Reach.	Cov.
preT-PPN	?	D	?	D
postT-PPN	?	D	?	D
PPN	U	U	U	U
distinctT-PPN	?	?	?	?
P-PPN	?	D	D	D

Summary for the Synthesis

Synthesis

- Given a marked preT-PPN $\mathcal{S}$ and a marking m, we can compute a finite representation of $\mathcal{CV}(\mathcal{S}, m)$.
- Given a marked postT-PPN $\mathcal{S}$ and a marking m, we can compute a finite representation of $\mathcal{CV}(\mathcal{S}, m)$.

Decidability in DistinctT-PPNs

Using monotonicity:
U-cov in distinct $\Leftrightarrow$ U-cov in the post where every paramaters on input arc evaluated to 0
E-cov in distinct $\Leftrightarrow$ E-cov in the pre where every paramaters on output arc evaluated to 0

New Results for DistinctT-PPNs

	$\mathscr{U}$-problem		$\mathscr{E}$-problem	
	Reach.	Cov.	Reach.	Cov.
preT-PPN	?	D	?	D
postT-PPN	?	D	?	D
PPN	U	U	U	U
distinctT-PPN	?	D	?	D
P-PPN	?	D	D	D

idea originally used for L/U-automata [Jovanović et al., 2015]

If it can be computed, the solution of the synthesis of coverability in distinctT-PPN cannot, in general, be represented using any formalism for which emptiness of the intersection with equality constraints is decidable.

What about Complexities ?

Hardness

Given a PN $\mathcal{S} \to$ build a PPN $\mathcal{S}'$ by adding an unused parameter

Existential or Universal coverability on $\mathcal{S}' \Leftrightarrow$ coverability on $\mathcal{S}$

Parametric Problems are at least EXPSPACE-hard

First Easy results

Coverability [Rackoff, 1978]

Coverability in PN is ExpSpace.

U-cov in postT-PPN

Given $\mathcal{S}$ a postT-PPN:

m U-cov in $\mathcal{S} \Leftrightarrow m$ coverable in $\mathbf{0}(\mathcal{S})$

E-cov in preT-PPN

Given $\mathcal{S}$ a pre-PPN:

m E-cov in $\mathcal{S} \Leftrightarrow m$ coverable in $\mathbf{0}(\mathcal{S})$

Complexities

E-cov for preT-PPN and U-cov for postT-PPN are ExpSpace-complete.

ωPN Semantics [Geeraerts et al., 2015]

Given a marking m, and a transition t such that $m \geq Pre(t)$, firing t from m gives a new marking m' s.t. $\forall p \in P, m'(p) = m(p) - Pre(p,t) + o$ where $o = Post(t,p)$ if $Post(p,t) \in \mathbb{N}$ and $o \geq 0$ if $Post(p,t) = \omega$. We denote this by $m \xrightarrow{t} m'$. Thus $Post(p,t) = \omega$ means that an arbitrary number of tokens are generated in p.

Coverability [Geeraerts et al., 2015]

Coverability in ωPNs is EXPSPACE-complete.

From postT-PPNs to ωOPNs

$\mathcal{N}$ a postT-PPN and $\mathcal{N}'$ the ωOPN where the parameters have been replaced by ω's.
Given $m \in \mathcal{RS}(\mathcal{N}', m_0)$, there exists a valuation v such that there exists a marking $m' \geq m$ with $m' \in \mathcal{RS}(v(\mathcal{N}), m_0)$.
Moreover, $\cup_{v \in \mathbb{N}^{\mathbb{P}}} \mathcal{RS}(v(\mathcal{N}), m_0) \subseteq \mathcal{RS}(\mathcal{N}', m_0)$.

Complexity of Existential Coverability

E-cov on postT-PPNs is ExpSpace-complete.

U-COV IN PRET-PPNS: STRATEGY

We address the problem of universal coverability through that of the more general universal simultaneous unboundedness. We will prove that both are EXPSPACE-complete.

EXPSPACE completeness of U-Cov in PreT-PPNs

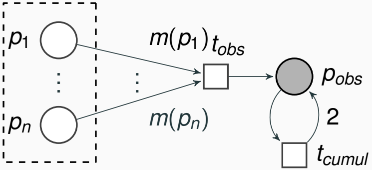
Simultaneous Unboundedness [Demri, 2013]

$X \subseteq P$, $\mathcal{S}$ is simultaneously X-unbounded if for any $B \geq 0$, there is a run w such that $m_0 \xrightarrow{w} m$ and for $i \in X$, we have $m(i) \geq B$.

[Demri, 2013]

The simultaneous unboundedness problem for Petri Nets is ExpSpace-complete.

Reduction of coverability to place boundedness



Intuition : skeleton of a ”good“ run

- U-simultaneous X unboundedness

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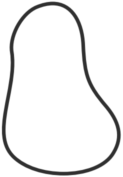
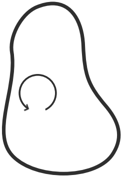
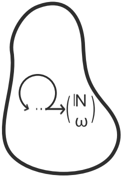
- We thus suppose we can guess this sequence of distinct parametric transitions $\sigma = \theta_1 \theta_2 \ldots \theta_l$
- The net should be universally simultaneous X unbounded Plus each input place of a parametric arc of one of the θ_i's should be universally unbounded.
- What matter is the order of the first occurence of each parametric transition involved.



N_p all parametric transitions removed

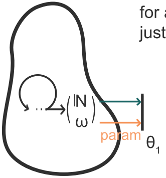


loops allowing to store enough tokens in the input places of the parametric arcs of θ_1



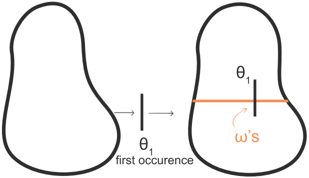
in terms of acceleration its creates some ω's

Intuition : toward a simpler property

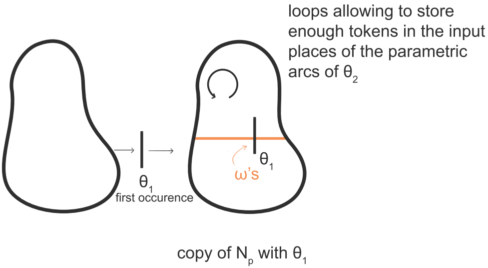


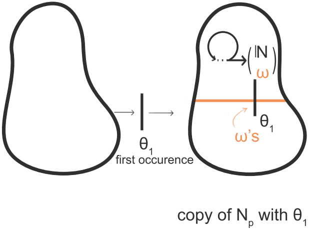
for a greater valuation
just repeat the loop...

Intuition : toward a simpler property

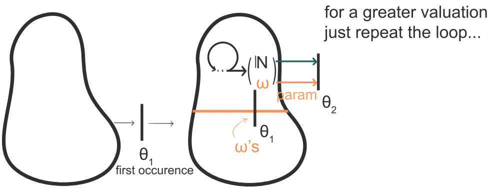


copy of N_p with θ_1


loops allowing to store enough tokens in the input places of the parametric arcs of θ_2
θ_1
first occurence
θ_1
ω's
copy of N_p with θ_1

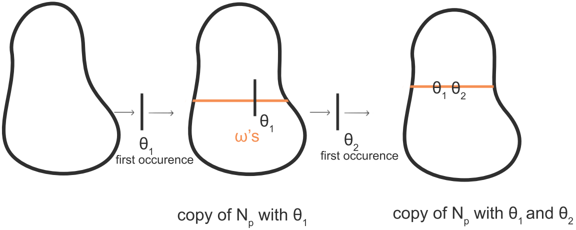

θ1
first occurence
IN
ω
θ1
ω's
copy of Np with θ1

Intuition : toward a simpler property

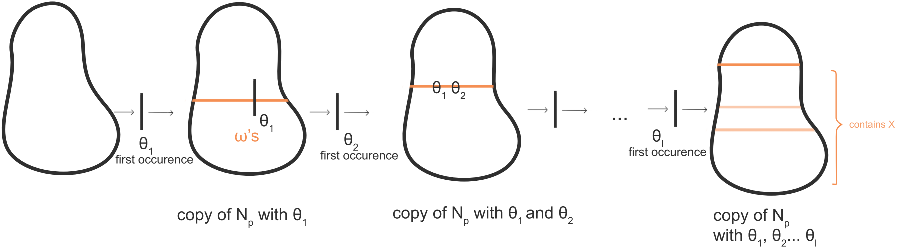


copy of N_p with θ_1

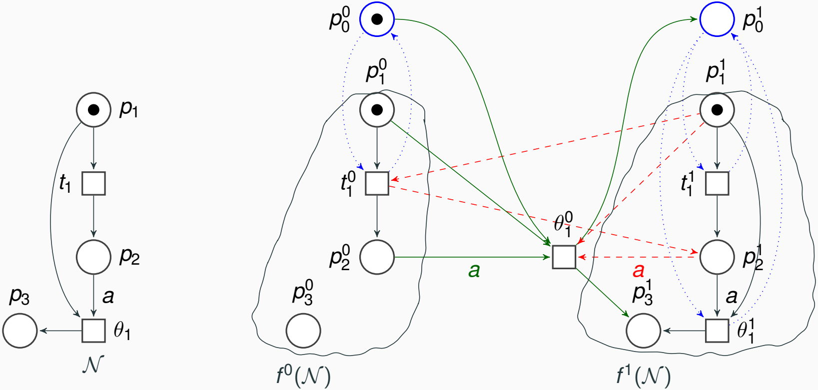
Intuition : toward a simpler property



INTUITION : TOWARD A SIMPLER PROPERTY



Incremental Net



Incremental Net $\mathcal{I}$ of $\mathcal{N}$ along $\sigma = \theta_1$

U-simultaneous Unboundedness in preT-PPNs : main result

Reduction of U-simultaneous Unboundedness to Simultaneous unboundedness

$\mathcal{N} = (P, T', Pre, Post, \mathbb{P})$ a preT-PPN
$T' = T \cup \Theta$ where:
Θ represents the parametric transitions of $\mathcal{N}$
T its plain transitions
Given X a set of places of P, the following propositions are equivalent:

1. $(\mathcal{N}, m_0)$ is universally simultaneously X unbounded
2. $\exists \sigma = t_1, ..., t_l$ a sequence of distinct parametric transitions, considering the incremental model $\mathcal{I}$ of $\mathcal{N}$ along σ, $(\mathbf{k}(\mathcal{I}), \mu_0)$ is simultaneously $f^l_{\mathcal{N}\to\mathcal{I}}(X) \cup \left(\bigcup_{t_i \in \sigma} f^{i-1}_{\mathcal{N}\to\mathcal{I}}(\Pi(t_i))\right)$ unbounded.

Complexity of U simult unboundedness in PreT-PPNs

The Universal Simultaneous Unboundedness problem for preT-PPNs is ExpSpace-complete.

Decision Results Enhanced with Complexities

	$\mathscr{U}$-problem		$\mathscr{E}$-problem	
	Reach.	Cov.	Reach.	Cov.
preT-PPN	?	ExpSpace-c	?	ExpSpace-c
postT-PPN	?	ExpSpace-c	?	ExpSpace-c
PPN	U	U	U	U
distinctT-PPN	?	ExpSpace-c	?	ExpSpace-c
P-PPN	?	ExpSpace-c	D	ExpSpace-c

Establishing Frontiers - Work In Progress

What was Previously Established...

	$\mathscr{U}$-problem			$\mathscr{E}$-problem		
	Reach.	S.Unbound	Cov.	Reach.	S.Unbound	Cov.
preT-PPN	?	?	D	?	?	D
postT-PPN	?	?	D	?	?	D
PPN	U	?	U	U	?	U
distinctT-PPN	?	?	D	?	?	D
P-PPN	?	?	D	D	?	D

What is Currently Established...

	$\mathscr{U}$-problem			$\mathscr{E}$-problem		
	Reach.	S.Unbound	Cov.	Reach.	S.Unbound	Cov.
preT-PPN	?	D	D	?	?	D
postT-PPN	?	?	D	?	?	D
PPN	U	U	U	U	U	U
distinctT-PPN	?	?	D	?	?	D
P-PPN	?	?	D	D	?	D

Monotonicity Technics: Evaluate to 0

	$\mathscr{U}$-problem			$\mathscr{E}$-problem		
	Reach.	S.Unbound	Cov.	Reach.	S.Unbound	Cov.
preT-PPN	?	D	D	?	D	D
postT-PPN	?	D	D	?	?	D
PPN	U	U	U	U	U	U
distinctT-PPN	?	?	D	?	?	D
P-PPN	?	D	D	D	?	D

Adapting Karp and Miller

	$\mathscr{U}$-problem			$\mathscr{E}$-problem		
	Reach.	S.Unbound	Cov.	Reach.	S.Unbound	Cov.
preT-PPN	?	D	D	?	D	D
postT-PPN	?	D	D	?	D	D
PPN	U	U	U	U	U	U
distinctT-PPN	?	?	D	?	?	D
P-PPN	?	D	D	D	D	D

Can be obtained by adapting KM procedure using a special acceleration involving acceleration:

- ω *iff* unbounded number of tokens
- $*$ *iff* arbitrary large (*i.e.*parameterised) but finite number of tokens

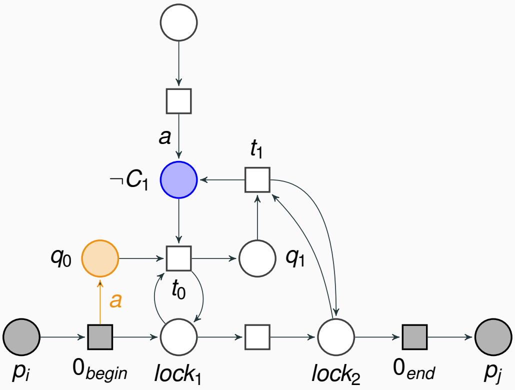
Classic Extention to DistinctT-PPNs

	$\mathscr{U}$-problem			$\mathscr{E}$-problem		
	Reach.	S.Unbound	Cov.	Reach.	S.Unbound	Cov.
preT-PPN	?	D	D	?	D	D
postT-PPN	?	D	D	?	D	D
PPN	U	U	U	U	U	U
distinctT-PPN	?	D	D	?	D	D
P-PPN	?	D	D	D	D	D

What about Reachability ?

	$\mathscr{U}$-problem			$\mathscr{E}$-problem		
	Reach.	S.Unbound	Cov.	Reach.	S.Unbound	Cov.
preT-PPN	?	D	D	?	D	D
postT-PPN	?	D	D	?	D	D
PPN	U	U	U	U	U	U
distinctT-PPN	?	D	D	?	D	D
P-PPN	?	D	D	D	D	D

LET US DO A 0 TEST WITH POSTT-PPNS...



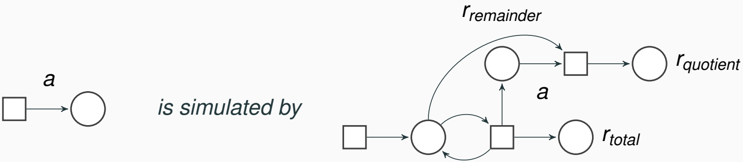
Cheat the zero test: fire 0_{begin} to 0_{end} when $k < a$ tokens in $\neg C_1$

Control Cheating: at least $a - k$ tokens will then be trapped in q_0

- not mandatory to refill $\neg C_1$ with all the tokens consumed $\Rightarrow q_1$ will be positive, but if the zero test occurring in the execution is fair, then it is possible to empty q_0.
- if this zero test is used several time in the machine:
    - if the zero test was faire previously, $\Rightarrow$ there was a run which leads to 0 tokens in both q_0 and q_1 so the construction was reseted.
    - if the zero test was used but not fair, $\Rightarrow$ some tokens are stored in q_0, say $h > 0$. Then, those tokens remain trapped in q_0, indeed, $\neg C_1$ has at most a tokens, and each time the zero test is involved, exactly a new tokens are generated in q_0, so there is no possibility to consume more than a tokens from q_0.

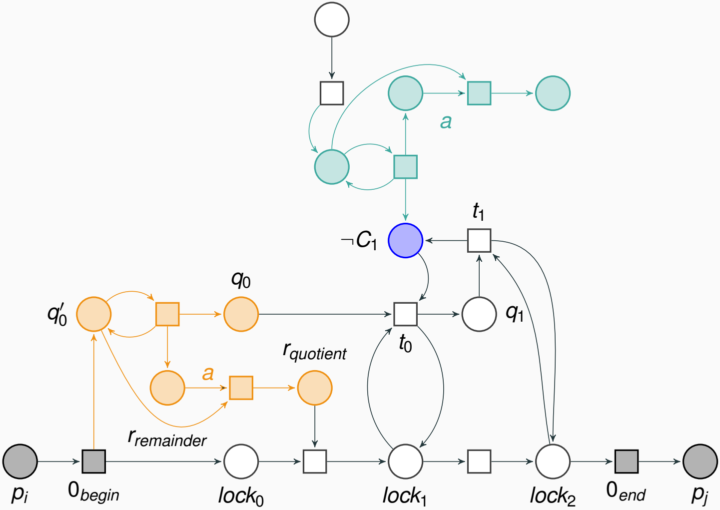
	$\mathscr{U}$-problem			$\mathscr{E}$-problem		
	Reach.	S.Unbound	Cov.	Reach.	S.Unbound	Cov.
preT-PPN	?	D	D	?	D	D
postT-PPN	U	D	D	U	D	D
PPN	U	U	U	U	U	U
distinctT-PPN	?	D	D	?	D	D
P-PPN	?	D	D	D	D	D

LET US GENERATE *a* TOKENS WITH A PRET-PPNS...



$$r_{total} = r_{quotient} \times a + r_{remainder}$$

LET US DO A 0 TEST WITH PRET-PPNS...



- to initialize the place $\neg C1$, indeed, with this construction, it is possible to generate an arbitrary number of tokens in $\neg C1$, nevertheless, $\neg C_1 = a$ iff the two places corresponding to the remainder and the quotient contain respectively 0 and 1 token.
- if the zero test can be fairly performed
  $\Rightarrow$ there is a run that leaves exactly 0 token in the *remainder* and in q_0 and q_1 but 1 token in *quotient*.

- if the zero test is used not fairly:
  - a tokens or more are created in q_0, then it is possible to consume only a tokens, thus some tokens will remain trapped in q_0
  - cheat by generating directly less than a tokens, same as postT-PPNs proof
  - Let us now imagine we perform this zero test two times, the first time is fair but more than a tokens are created, say $a + k$. Then k tokens remains in q_0 and the place $r_remainder$ after the firing of this test. Now, let us suppose we perform an unfair zero test, that is to say less than a tokens are in $\neg C_1$. We could generate only $a - k$ tokens in $r_remainder$ such that now q_0 as a tokens. Nevertheless, we obtain a tokens in q_0. Thus tokens are trapped in q_0.

	$\mathscr{U}$-problem			$\mathscr{E}$-problem		
	Reach.	S.Unbound	Cov.	Reach.	S.Unbound	Cov.
preT-PPN	U	D	D	U	D	D
postT-PPN	U	D	D	U	D	D
PPN	U	U	U	U	U	U
distinctT-PPN	?	D	D	?	D	D
P-PPN	?	D	D	D	D	D

Classic Extention to DistinctT-PPNs

	$\mathscr{U}$-problem			$\mathscr{E}$-problem		
	Reach.	S.Unbound	Cov.	Reach.	S.Unbound	Cov.
preT-PPN	U	D	D	U	D	D
postT-PPN	U	D	D	U	D	D
PPN	U	U	U	U	U	U
distinctT-PPN	U	D	D	U	D	D
P-PPN	?	D	D	D	D	D

Conclusion

Summary

Synthesis preT

Given a marked preT-PPN $\mathcal{S}$ and a marking m, we can compute a finite representation of $\mathcal{CV}(\mathcal{S}, m)$.

Synthesis postT

Given a marked postT-PPN $\mathcal{S}$ and a marking m, we can compute a finite representation of $\mathcal{CV}(\mathcal{S}, m)$.

Future Work ?

	$\mathscr{U}$-problem			$\mathscr{E}$-problem		
	Reach.	S.Unbound	Cov.	Reach.	S.Unbound	Cov.
preT-PPN	U	D	D	U	D	D
postT-PPN	U	D	D	U	D	D
PPN	U	U	U	U	U	U
distinctT-PPN	U	D	D	U	D	D
P-PPN	?	D	D	D	D	D

Future Work ?

	$\mathscr{U}$-problem			$\mathscr{E}$-problem		
	Reach.	S.Unbound	Cov.	Reach.	S.Unbound	Cov.
preT-PPN	U	ExpSpace-c	ExpSpace-c	U	ExpSpace-c	ExpSpace-c
postT-PPN	U	ExpSpace-c	ExpSpace-c	U	D	ExpSpace-c
PPN	U	U	U	U	U	U
distinctT-PPN	U	ExpSpace-c	ExpSpace-c	U	D	ExpSpace-c
P-PPN	?	ExpSpace-c	ExpSpace-c	D	D	ExpSpace-c

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Questions?
Remarks?
Advices?