A Random Testing Approach using Pushdown Automata

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Outline

Random Exploration

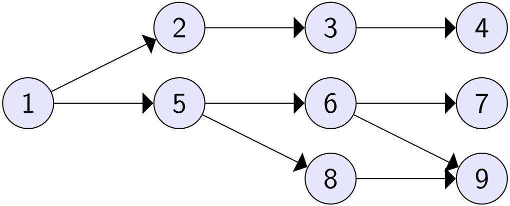
Random exploration optimizes the coverage...

- Robustness testing, Fuzz testing,
- Performance testing,
- Search based testing,
- Combination of random exploration with other techniques

Combination of graph-based testing and coverage criteria.

Alain Denise, Marie-Claude Gaudel, Sandrine-Dominique Gouraud, Richard Lassaigne, Johan Oudinet, and Sylvain Peyronnet. Coverage-biased random exploration of large models and application to testing. *STTT, 2012*.

Testing on Graphs



All states,
all transitions,
all loops, etc.

Huge graphs lead to a huge number of tests.
Optimization of the number of tets can be hard (NP-complete).

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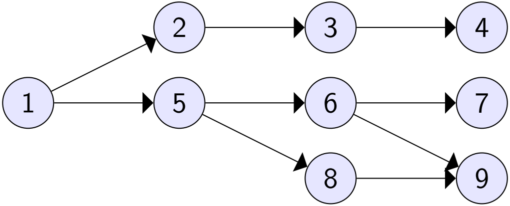
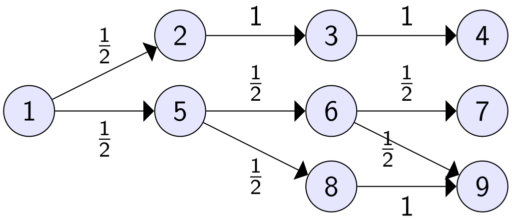


All states,
all transitions,
all loops, etc.

Huge graphs lead to a huge number of tests.
Optimization of the number of tets can be hard (NP-complete).
Reduce the number of tests using a random approach.

Isotropic Random Walks



Paths of length 3.

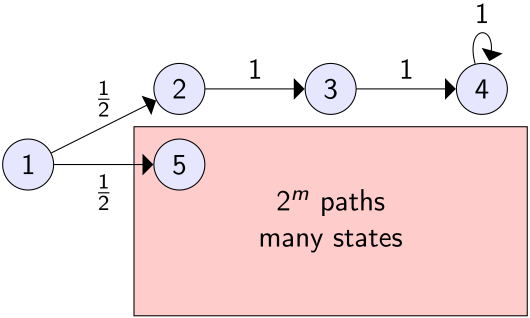
$1 \to 2 \to 3 \to 4$ probability $\frac{1}{2}$.

$1 \to 5 \to 6 \to 7$ probability $\frac{1}{8}$.

$1 \to 5 \to 6 \to 9$ probability $\frac{1}{8}$.

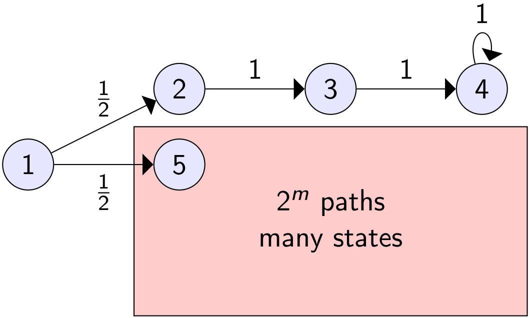
$1 \to 5 \to 8 \to 9$ probability $\frac{1}{4}$.

Isotropic Random Walks



$1 \to 2 \to 3 \to 4 \to \ldots \to 4$ probability $1/2$.
Other paths, probability $1/2^{m+1}$.

Isotropic Random Walks



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Over representation of states 2, 3, 4 and related transitions

Local decisions don't provide a fair global coverage.

Uniform Generation of Paths

Each path of length n is picked up with the same probability.

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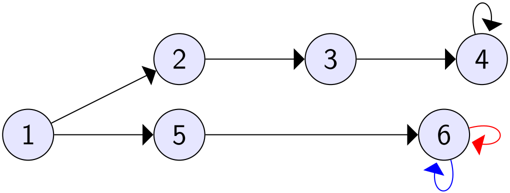
Each path of length n is picked up with the same probability.
But some states/transitions can be visited by many paths of length n.



Prob. to cover 2: $\frac{1}{1+2^{n-2}}$

Prob. to cover 5: $\frac{2^{n-2}}{1+2^{n-2}}$

In this case the random walk approach provides a better state coverage.

Quality of the Random Process

Probabilistic quality [T-FW89]

Given a random generation process, its quality $q_{C,N}$, relatively to a coverage criterion C, is the probability that all elements of C are covered when generating N test cases.

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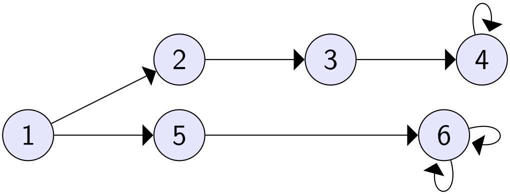

Unif. Gene. Paths.

$q_{\mathrm{states},N} = 1 - (1 - \frac{1}{1+2^{n-2}})^N$

Not good!

Random Walk

$q_{\mathrm{states},N} = 1 - \frac{1}{2^N}$

Very good!

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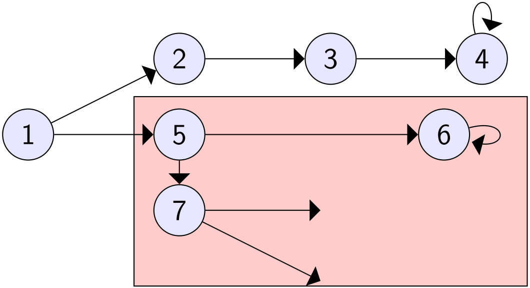

Unif. Gene. Paths
(Very) Good!

Random Walk
Not good!

Random biased Generation

Algorithm [GDG+01]

Repeat N times:

1. Generate $c \in C$ randomly with probability π_c
2. Generate uniformly a path of length n visiting c.

[DGG+12]: how to compute π_c to optimize the quality.

Maximize $q_{\min}$ satisfying

$$\begin{cases} q_{\min} \leq \sum_{c' \in C} \mathrm{prob}(c, c')\pi_c & \text{for all } c \in C \\ \sum_{c \in C} \pi_c = 1 & \\ 0 \leq \pi_c \leq 1 & \text{for all } c \in C \end{cases}$$

prob(c,c'): probability that a path of length n visits both c and c'.

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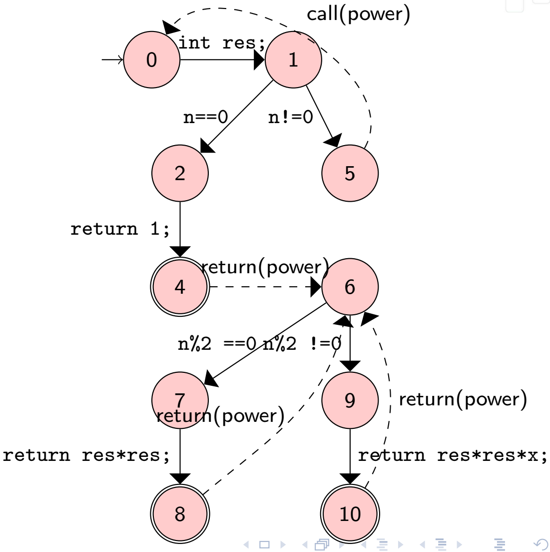
Requirements:

a. Computing the probability that a path visits both c and c'.
b. Solving a linear programming problem with $|C|$ variables.
c. Doing 2.

[DGG+12]: for graphs, for the state coverage criterion.

The Concretization Problem

```
int power(float x, int n){
    int res;
    if (n==0) {
        return 1;
    } else {
        res = power(x,n/2);
        if (n%2==0) {
            return res*res ;
        } else {
            return res*res*x
;
        }
    }
}
```



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power(x,1)
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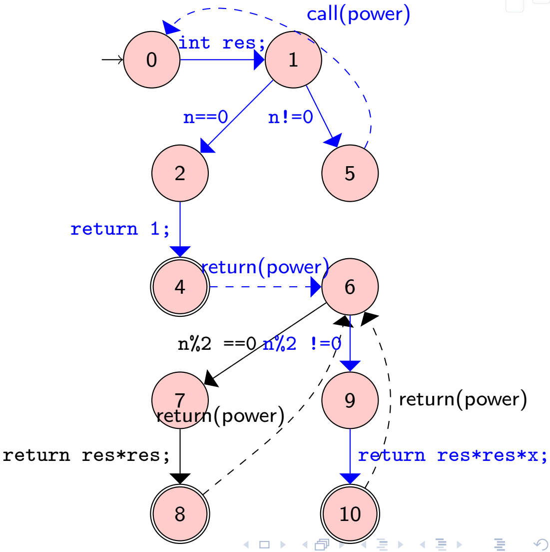

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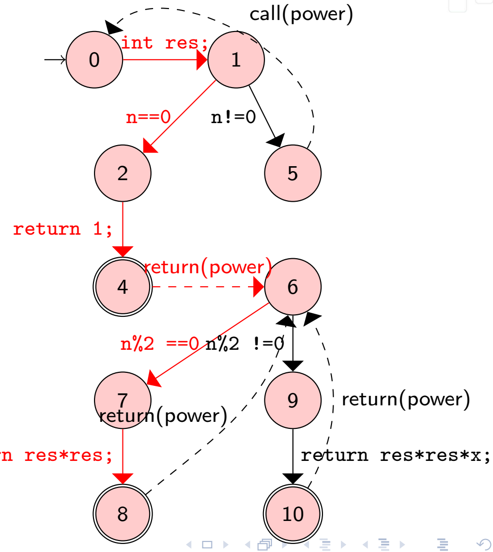


This path cannot be concretized.

Motivations and Contributions

Main Issue

1. To avoid the concretization problem, use data flow graphs: a huge number of states (billions for simple programs).
2. Random biased approaches require quite small C (and therefore quite small graphs).

Goal

Develop similar techniques for graphs carrying information.

Our Work

Pushdown automata and state/transition coverage criteria.

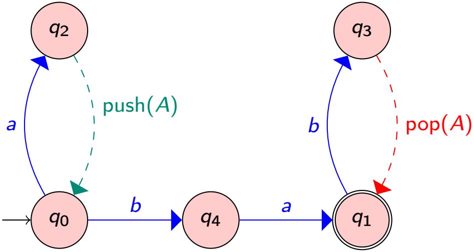
Outline

Modelling Systems using Pushdown Automata

Pushdown automata (PDA)

- Polynomial time decidability of emptiness, membership,
- Useful to model systems with (mutually) recursive functions,
- Useful to model parsing algorithms,
- Automatic tools to transform Java or C code into pushdown automata,
- Efficient model-checking tools.

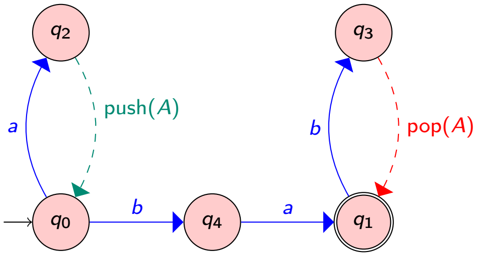
We consider Normalized Deterministic Pushdown Automata.



$\Gamma = \{A\}$ (stack)
$\Sigma = \{a, b\}$ (actions)
$Q = \{q_0, \dots, q_4\}$ (states)
$\Delta = \{(q_0, a, q_2), (q_2, \mathrm{push(A)}, q_0), (q_3, \mathrm{pop(A)}, q_1), \dots\}$ (transitions)

Each transition is labelled either by an action or a pop action or a push action.

Configurations in PDA



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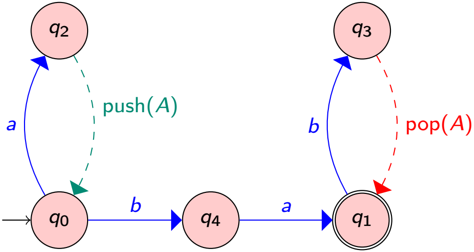

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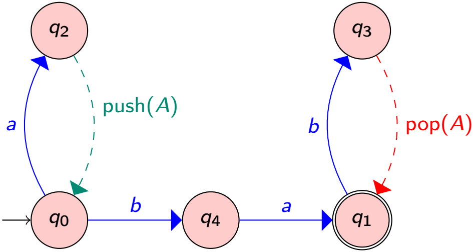

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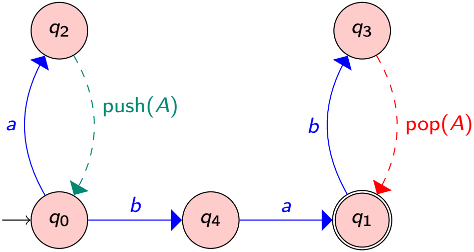
Configurations in PDA



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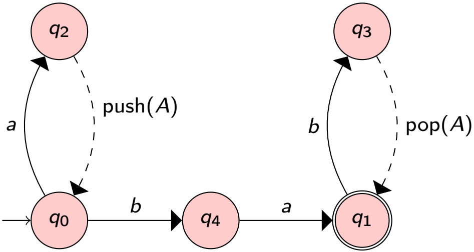
Successful PDA-paths



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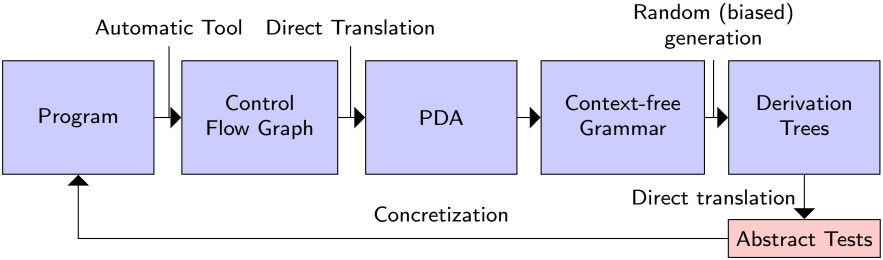
A *successful PDA-path* is a sequence of configurations linked by transitions, with initial and final conditions.

$$(q_0, \varepsilon) \rightarrow_a (q_2, \varepsilon) \rightarrow_{\mathrm{push(A)}} (q_0, A) \rightarrow_b (q_4, A) \rightarrow_a (q_1, A) \rightarrow_b (q_3, A) \rightarrow_{\mathrm{pop(A)}} (q_1, \varepsilon)$$

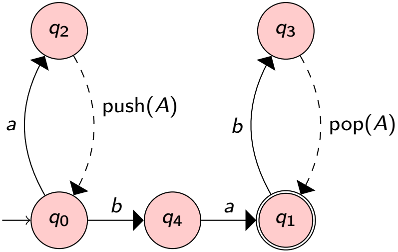
Given a path of length $4n + 2$ in the graph from q_0 to q_1, the probability that it corresponds to a successful PDA-path is $\frac{1}{2n+1}$.

Outline

Overview of the Approach (Doing 2.)

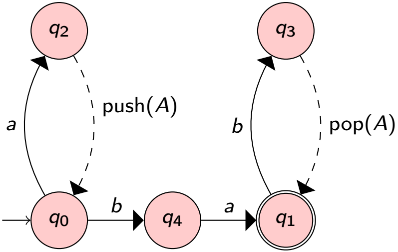


From Pushdown Automata to Grammars



From Pushdown Automata to Grammars

One can compute a grammar generating successful paths.



$$X_0 \to (q_0, a, q_2)X_2 \mid (q_0, b, q_4)(q_4, a, q_2)$$
$$X_2 \to (q_2, \mathrm{push(A)}, q_0)X_0S$$
$$S \to (q_1, b, q_3)(q_3, \mathrm{pop(A)}, q_1)$$

From Pushdown Automata to Grammars

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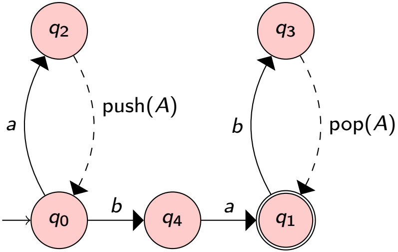

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Bijection Paths/D. Trees.

$$X_0 \to (q_0, a, q_2)X_2 \to (q_0, a, q_2)(q_2, \mathrm{push(A)}, q_0)X_0S$$
$$\to^* [(q_0, a, q_2)(q_2, \mathrm{push(A)}, q_0)]^2 X_0SS$$
$$\to [(q_0, a, q_2)(q_2, \mathrm{push(A)}, q_0)]^2(q_0, b, q_4)(q_4, a, q_2)SS$$
$$\to^* [(q_0, a, q_2)(q_2, \mathrm{push(A)}, q_0)]^2(q_0, b, q_4)(q_4, a, q_2)[(q_1, b, q_3)(q_3, \mathrm{pop(A)}, q_1)]^2$$

A Counting Approach [NW78][FZC94]

Example (a, b, c, d are terminal symbols):

$$E := EcE \mid EEd \mid a \mid b$$

The size of a derivation tree is its number of leaves.

- c_n number of trees of size n whose root rule is $E \to EcE$.
- d_n number of trees of size n whose root rule is $E \to EEd$.
- a_n number of trees of size n whose root rule is $E \to a$.
- b_n number of trees of size n whose root rule is $E \to b$.
- v_n number of trees of size n, $v_n = a_n + b_n + c_n + d_n$.

One has

- $v_1 = 2$, $v_2 = 0$ and $v_3 = 8$ $\qquad (a|b)c(a|b)$ and $(a|b)(a|b)d$.
- For example,

$$c_n = \sum_{k=1}^{n-1} v_k v_{n-k-1}.$$

- v_n, a_n, b_n, c_n and d_n can be computed recursively.

Random Generation of Trees [NW78][FZC94]

$E := EcE \mid EEd \mid a \mid b$ To generate derivation trees of size n.

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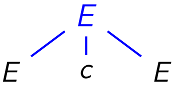
1. Compute a_k, d_k, b_k, c_k, v_k for all $k \leq n$.

Random Generation of Trees [NW78][FZC94]

$E := EcE \mid EEd \mid a \mid b$ To generate derivation trees of size n.

1. Compute a_k, d_k, b_k, c_k, v_k for all $k \leq n$.
2. Root rule is
   $E \rightarrow EcE$ with probability $\frac{c_n}{v_n}$,
   $E \rightarrow EEd$ with probability $\frac{d_n}{v_n}$,
   $E \rightarrow a$ with probability $\frac{a_n}{v_n}$ and
   $E \rightarrow b$ with probability $\frac{b_n}{v_n}$.



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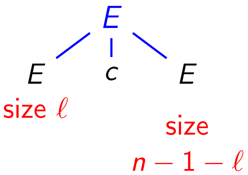
3. Left child has size ℓ with probability $\frac{v_\ell v_{n-1-\ell}}{c_n}$.



Note that v_n is the number of PDA-paths of length n.

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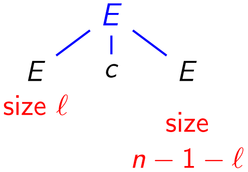
4. Recursive generation.



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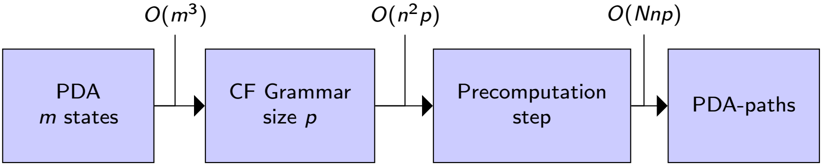
Complexity

Generation of N PDA-paths of length n.



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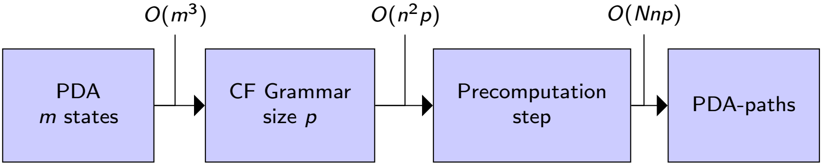

Cleaning step

Random biased Generation (Reminder)

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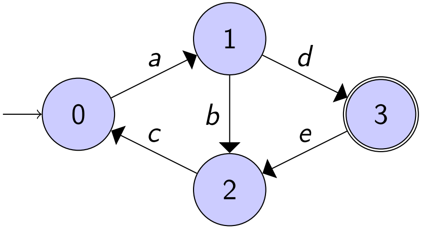
Random biased Generation for States



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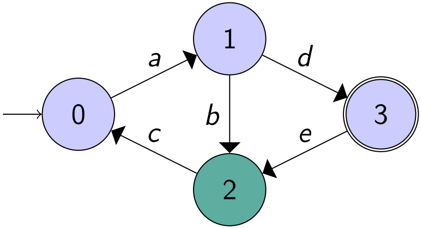

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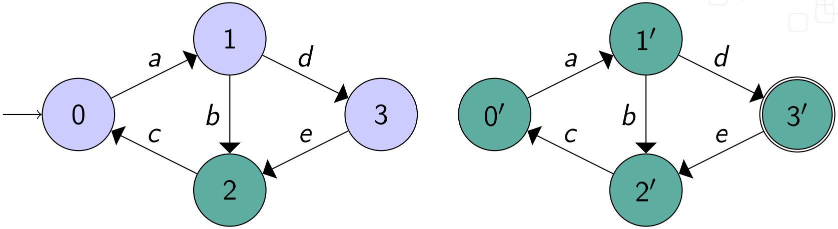

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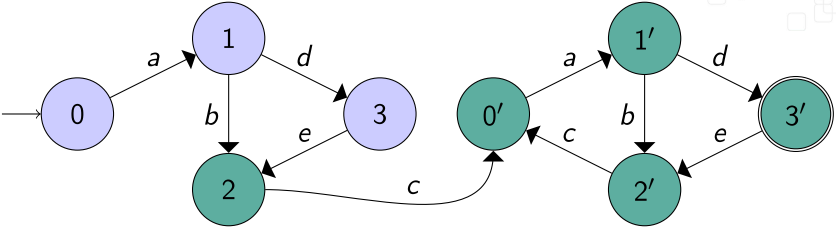
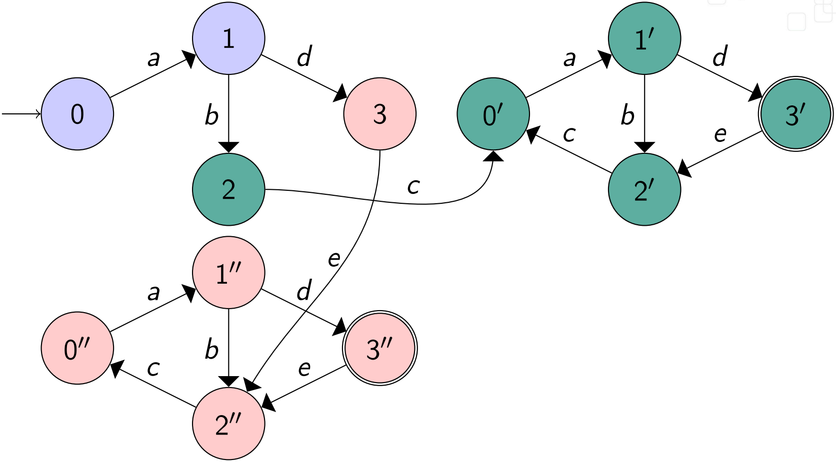


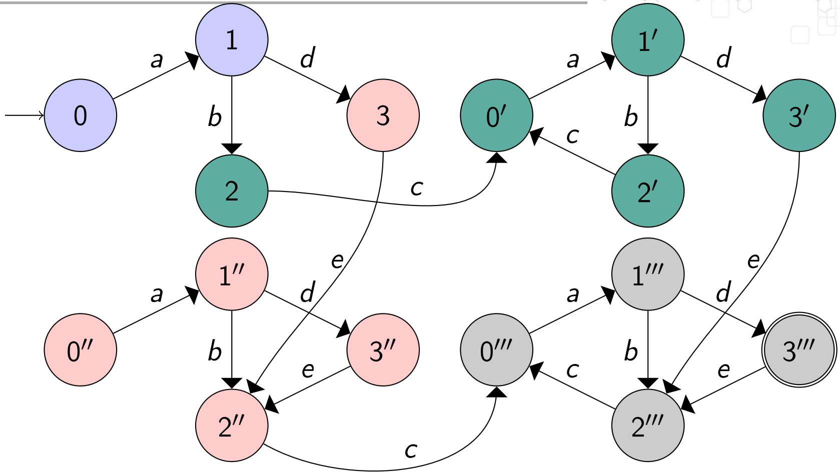
Allows the generation (and the enumeration) of paths of length n visiting 2.

Random biased Generation for States



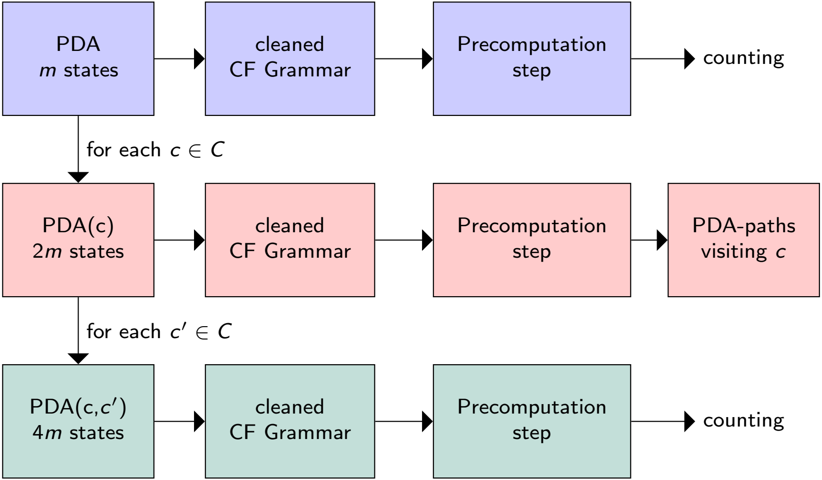
Allows the generation (and the enumeration) of paths of length n visiting 2 or 3.

Random biased Generation for States



Allows the generation (and the enumeration) of paths of length n visiting 2 and 3. Similar construction for transitions.

Overview



PDA
m states
cleaned
CF Grammar
Precomputation
step
counting
for each $c \in C$
PDA(c)
$2m$ states
cleaned
CF Grammar
Precomputation
step
PDA-paths
visiting c
for each $c' \in C$
PDA(c,c')
$4m$ states
cleaned
CF Grammar
Precomputation
step
counting

Using Coverage Criteria

Algorithm 1

Randomly and uniformly generates PDA-paths until C is covered.

Algorithm 2

Randomly and uniformly generates a PDA-path visiting a non already covered element until C is covered.

Algorithm 3

Randomly generates a PDA-paths with optimized probabilities element until C is covered.

Outline

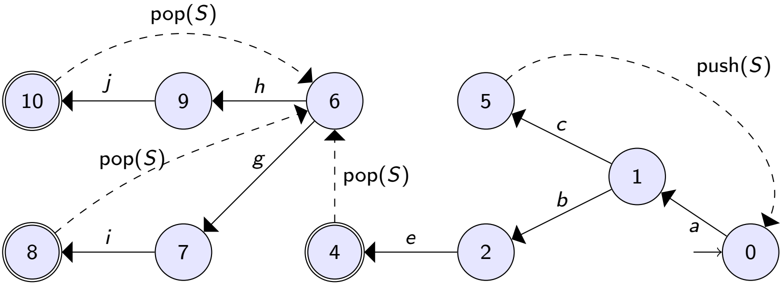
Examples, Shunting Yard Algorithm

Algorithm to translate an arithmetic expression into the equivalent expression in the Reverse Polish notation.

$$(3+2)*(5+4) \quad \rightarrow \quad 3\ 2\ +\ 5\ 4\ +\ *$$

The algorithm exploits a stack data structure.

Examples, the Power Function



Corresponding to the control flow graph of function computing x^k.

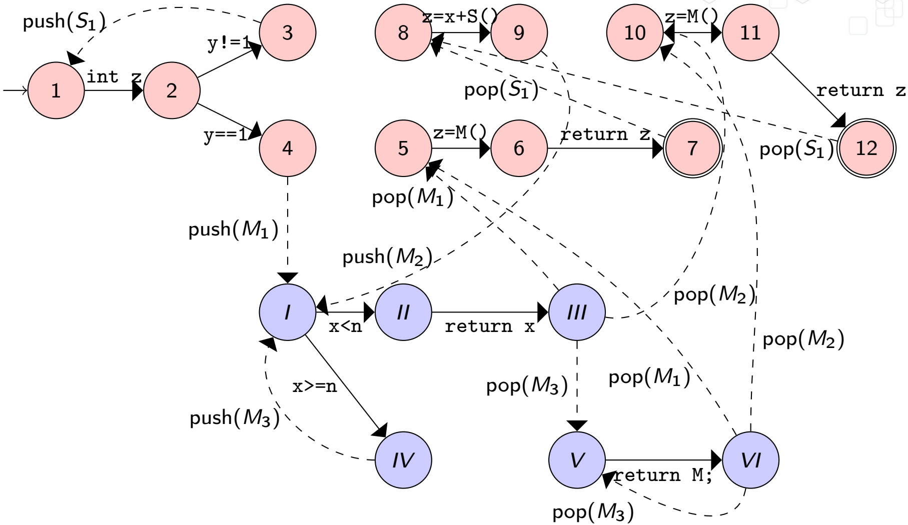
Examples, the Modulo Function

$M(x, n) = x$ modulo n and $S(x, y, n) = x * y$ modulo n

```
int S(int x, int y, int
n){
    int z;
    if (y == 1){
        z = M(x,n);
        return z;
    } else {
        z = x +
S(x,y-1,n);
        z = M(z,n);
        return z;
    }
}
```

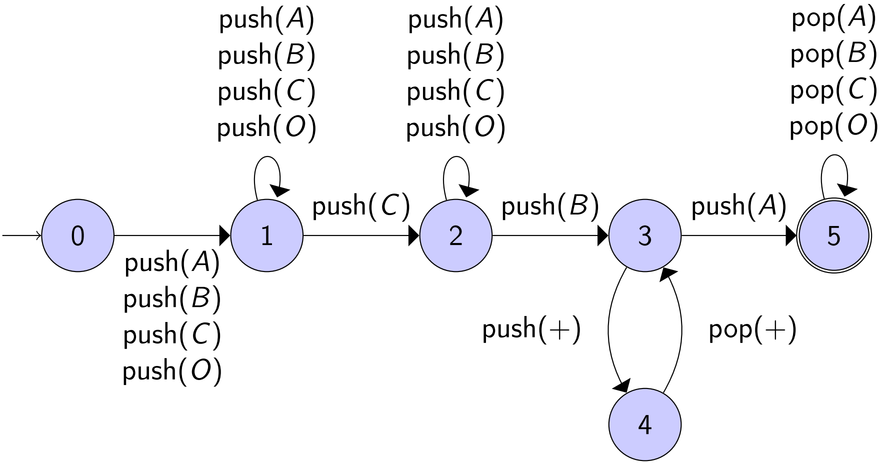
```
int M(int x, int n){
    if (x < 1){
        return x;
    } else {
        return M(x-n,n);
    }
}
```

Examples, the Modulo Function



Examples, XPATH Query

Modeling an XPATH query (from [C08]).



Examples, Sizes of the PDA

	states	transitions (1)	transitions (2)
Shunting Yard	23	36	102
Power	10	12	24
Modulo	18	24	90
XPATH	6	21	102

(1) Normalized transitions
(2) Considering all transitions

Sizes of the examples are comparable to those obtained by the automatic transformation into pushdown models of some industrial drivers [ST12].

NPDA vs. Finite Automata: $\mathcal{A}_{\text{modulo}}$

Path size	Our approach	Approach of [DGG+12]
from 1 to 7	no NPDA-trace	no DFA-trace
8	1-2-4-I-II-III-5-6-7 (10)	1-2-4-I-II-III-10-11-12 (6) 1-2-4-I-II-III-5-6-7 (4)
9	no NPDA-trace	no DFA-trace
10	no NPDA-trace	1-2-4-I-IV-I-II-III-5-6-7 (3) 1-2-4-I-IV-I-II-III-10-11-12 (2) 1-2-4-I-II-III-V-VI-10-11-12 (3) 1-2-4-I-II-III-V-VI-5-6-7 (2)
11	no NPDA-trace	A-2-3-1-2-4-I-II-III-10-11-12 (6) 1-2-31-2-4-I-II-III-5-6-7 (4)

NPDA vs. Finite Automata: $\mathcal{A}_{\text{modulo}}$

n	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
NPDA	1	0	0	0	1	0	0	1	1	0	0	1	1	0	0
DFA	6	4	10	6	18	10	34	18	64	34	114	64	200	114	356

n	27	28	29	30	31	32	33	34	35	36	37	80
NPDA	3	1	0	1	4	1	0	3	5	1	0	142
DFA	200	640	356	1152	640	2066	1152	3692	2066	6598	3692	$2.4\ 10^9$

For several lengths, as 21, 29, 37, there is no NPDA-traces. All generated DFA-traces would be not consistent.

Mutant based Evaluation

Using the PDA model of the Shunting Yard Algorithm and its implementation (from Wikipedia).

Uniform random generation of PDA-paths of length n.

Number of required test cases (20 experiments for each case).

	Killing mutants			Covering transitions			Cov. 79% of the code		
n	min	max	aver.	min	max	aver.	min	max	aver.
10	2	16	7.8	7	48	15.3	3	17	6.9
15	1	7	3.7	4	19	8.8	2	7	4.5
20	2	11	3.2	3	10	5.4	2	6	2.9

10 different mutants used: 4 Inc./Dec., 1 Arr. Ref. Replacement, 1 Switch Statement Mut., 4 Log. Neg.

Uniform Random Generation

	Grammar Size	Grammar Gen. Time(s)	Cleaned Grammar	Grammar Cleaning Time(s)	Precomp. Time(s)	100 Paths Gen. Time (s)
$\mathcal{A}_{\mathrm{xpath}}$	217+ 3463	0.03	27+87	0.38	(10) 0.07 (20) 0.22 (30) 0.57 (100) 17.21 (200) 141	(10) 0.09 (20) 0.17 (30) 0.63 (100) 1.1 (200) 2.83
$\mathcal{A}_{\mathrm{power}}$	201+369	0.02	27+31	0.05	(100) 0.98 (200) 5.63 (500) 78.96	(100) 0.41 (196) 0.94 (496) 3.94
$\mathcal{A}_{\mathrm{modulo}}$	1621+ 7572	0.07	45+50	6.07	(10) 0.03 (20) 0.08 (50) 0.41 (100) 2.29 (200) 14.86	(8) 0.07 (21) 0.14 (49) 0.32 (99) 0.28 (199) 1.4
$\mathcal{A}_{\mathrm{SY}}$	1937+ 16735	0.13	143+ 261	16.61	(10) 0.16 (20) 0.58 (30) 1.46 (100) 39.21 (200) 326.28	(10) 0.13 (20) 0.24 (30) 0.34 (100) 1.14 (200) 2.35

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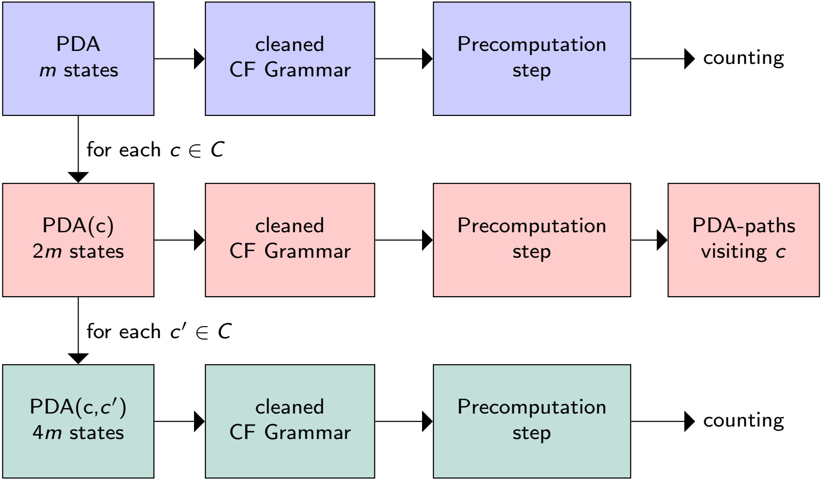
Overview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for each $c' \in C$
PDA(c,c')
$4m$ states
cleaned
CF Grammar
Precomputation
step
counting

Computation time for Algorithm 2/3

From cleaned grammars: Computation time for
1) generating paths of length 60 visiting a given state/transition, and
2) computing their probabilities.

	$\mathcal{A}_{\mathrm{xpath}}$	$\mathcal{A}_{\mathrm{power}}$	$\mathcal{A}_{\mathrm{modulo}}$	$\mathcal{A}_{\mathrm{SY}}$
total time (state)	38s	5.25s	22min	136min
av. grammar size	31.3+98.8	32.2+36.8	65.2+72.9	342.7+692.9
total time (transition)	91s	6.3s	33min	219min
av. grammar sizes	30.5+94.8	30.7+34.3	66.2+73.7	273.3+572.6

Computation time for Algorithm 3

Computation time of the linear programming systems.

	$\mathcal{A}_{\text{xpath}}$	$\mathcal{A}_{\text{power}}$	$\mathcal{A}_{\text{modulo}}$	$\mathcal{A}_{\text{SY}}$
av. time (one pair of states)	13.4s	0.9s	408s	110min
total (state)	7min19s	79s	34h12min	847h (*)
total (transition)	25min	81s	60h39min	1200h (*)
total (simplified, state)	1min	insign.	9h (*)	120h (*)
total (simplified, transition)	8min (*)	insign.	18h (*)	350h (*)

(*) estimated time.

Simplifications based on symmetry/replication.

Comparing Algorithms for $\mathcal{A}_{\mathrm{xpath}}$

Average number of paths for several lengths, to cover either all states or all transitions.

n	12	14	16	18	20	22	24	26	28	30	32
Algo. 1	7.51	6.1	5.0	4.77	4.74	4.24	4.43	4.37	4.34	4.34	4.33
Algo. 2	1.87	1.84	1.83	1.77	1.8	1.78	1.79	1.77	1.77	1.76	1.77
Algo. 3	1	1	1	1	1	1	1	1	1	1	1
Algo. 1	14.1	11.03	10.08	9.27	8.83	9.06	8.64	9.36	9.45	8.59	9.43
Algo. 2	6.9	6.07	5.57	5.08	4.92	4.68	4.66	4.71	4.51	4.52	4.51
Algo. 3	14.1	-	-	9.1	-	-	-	-	8.8	-	-

– Performing100 experiments

– Comparing theoretical and experimental results

Outline

Conclusion and Future Work

On computation times

1. Results obtained using a Python based prototype,
2. Many possible optimizations,
3. Can easily be distributed.

Conclusion

- Algorithms 1 and 2 are tractable.
- Challenging results.
- Algorithm 3 requires more investigations to handle large examples.

Future Work

Perspectives

- Look for Algorithm 3 optimizations,
- Develop an optimized tool,
- Develop efficient algorithms for cleaning the grammar,
- Develop similar approaches for other classes of automata (counter automata, timed automata, ...)